Academic Success Center

★Title III *Avenue to Success* Program

How to Solve Word Problems

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I know: you hate them, can't do them, they horrify you.
Now, what is so horrible about continuing to do something ***you have always done*** - ever since you began to understand and speak a language and to assign values to things? The truth is that **you have in you a method for solving word problems** already.

An Everyday Word Problem – Proof that You Already Know How

So, you say you do not believe my last statement? Well, please take a few moments to consider the following everyday scenario in which you must decide on some course of action.

> Your workplace is running a voluntary weekend workshop to benefit a local charitable organization. All employees are encouraged to participate. This, like any activity, involves your **time**, your **effort** and a certain **personal cost**, of which the latter may be ***quantifiable,*** or assigned a value, on the basis of the following:

- how many other things you would be missing on account of doing this activity
- financial losses
- emotional expenditure, etc.

Time is easily given a value by looking at your watch, and effort would be quantified by other determinations like those above:

- how much you sweat
- how many headaches this gives you
- how many books you must read to do the job, etc.

Let me suggest to you that you have begun, in this situation, a quick calculation to help you determine whether being involved in this project is feasible for you. If the foregoing has made sense to you, then you agree with the "***dimensional units***" or the factors involved in the equation, that is,

- **time, T**
- **effort, E**
- **personal cost, PC**

Your natural, intuitive problem-solving ability already has a method for placing these factors in a sensible relationship (***equation or formula***) which will yield the ***product***, or the answer you are after, namely, do I proceed with this activity or not? We could call the product **Personal Benefit, PB**, and we could write the equation as:

$$\frac{\mathbf{E \times T}}{\mathbf{PC}} = \mathbf{PB},$$

in which we see that, for us, the effort **E** we expend in this activity is ***directly proportional*** to the personal benefit we reap from it. That is, the more we exert ourselves, the more, by that degree of magnitude, will we obtain personally from this involvement. Since time **T** is also in the numerator, on the same plane with our product which is in the numerator on the other side, the same is true for time: that is, more time in, more benefit out. However, personal cost, **PC**, is ***inversely proportional*** to our product. You can see from the equation that the more you personally go into debt doing this project, that is, the greater becomes **PC**, the smaller will be personal benefit, **PB.**

Great; now then, how do we finally solve this dilemma? Well, again, your innate self-knowledge and values system will bring a kind of numerical stamp or quantity to each dimensional unit. These are the amounts of things that we "plug in" to the appropriate places in the equation.

Can you see now that you have been solving word problems forever? In your study of the sciences, you will continue to either be plagued or delighted by them. The word problem is the format for the problems presented to us in life, in social situations and, especially as we are ultimately concerned in your training here, in the workplace. Very few bosses, clients, patients, or situations speak to us in numbers or symbols.

Nonetheless, we must be able to translate their words or the concepts therein into mathematical
equations so that we can assign value and significance to these concepts, and so prioritize our tasks and get our jobs done. We pick up all the time on the code words, body language, tone of voice, and the rest which people use to convey quantity and magnitude. For example, wide open eyes or arms would mean a lot of something. If someone was quantifying pain, a deeply lined and pinched facial expression might mean a greater degree of pain than the wave of a hand. (And that all depends on the person who is pinching or waving.)

I have said all this so you will see that you know how to do word problems in an intuitive way. Now, there is more good news: with science word problems, there is far less interpretation than in everyday life situations! Quantities are given, never implied. Dimensional units may be implied to some extent, but in a language that is readily recognizable from consistent study of the subject matter. In other words, there are only a few ways to talk about the concept of gravity. Noone in a science text is going to wax poetic about apples falling on heads and expect you to read *"gravity."* Trust me; it is going to be much easier to do your homework than to sort out the real-life word problems you've been solving without ever knowing it. But you will have to pay attention to the new language of science you are learning so that you can pick up on the nuances of phrase that will be used to infer which terms are in play in the given problem.

Visualizing and Writing as You Read

My own, and many of my classmates', success with doing word problems hailed from us listening to our high school physics teacher's constant exhortation to:

WRITE AS YOU READ!!!!

I really cannot emphasize this technique enough. Do not bother reading the problem through once. You are wasting time. Whenever you do a word problem, have your pencil in hand, and plan on filling up the sheet of paper. As soon as you see a quantity, a dimension, recognize the equation you will need, feel the urge to sketch the situation, whatever, **write it then and there.** Do not hesitate. Do not hope that you will be struck by some kind of scientific lightning bolt that will bring life to the

primordial goo of your brain, and so the answer handed to you from above. Once again, science problems are not mysterious or 'tricky' as our life problems often seem to be. If you are befuddled by what appears to be a mysterious and insoluble problem, then you must go back to your notes or textbook section which deals with solving that sort of problem, and all will be revealed to you. I promise you that this is true. And, after that, **writing as you read will save you time and confusion.** Again, I promise.

Successful problem solvers are able to visualize the 'players' and their relationships in the problem. That is why I included the words *'sketch the situation'* above. As soon as you recognize a value, write it down. When you see the next one and you recognize a relationship, even if you don't have the formula in mind yet, indicate whatever relationship you recognize by sketching or in symbols (Tom Henderson, 1996-2005, The Physics Classroom Tutorial; see http://gbs.glenbrook.k12.il.us/Academics/gbssci/phys/chemphys/audhelp/probtips.html).

The All-Important Formula

Once you have an idea of the relationship of your problem players, you can now determine the type of formula you will need to manipulate to come up with the product. **A formula is nothing more than a scientific rule into which you fit your players, or *variables*.** Go back to our everyday example. Suppose things worked out well for you by applying that rule to that situation. Now, you are asked to do something else, somewhere else for someone else. You might be very likely to use the same formula, **E x T/PC = PB** to determine your course of action. This is the part of word problem solving which really tests your resourcefulness. Read the section of your textbook or notes which covers working the type of problem you are doing. Understanding the theory by means of using diagrams or talking it out or rewriting an outline is where you use your own learning style. It is to this process that you will attach the formula.

The next stumbling block you will find is that the formula does not automatically conform to your needs in any given problem. You must often rearrange the formula so that you have isolated the unknown variable, and all the rest of the variables for which you have numerical quantities are living on the other side, where you will crunch them together into one nugget. This is called ***algebraic manipulation***.

And Now, A Word About Algebra

In order to do word problems, you must be very comfortable with algebraic manipulations. This means that you must be able to get a standard equation to work *for you*. Remember our example of the volunteer activity, in which our equation yielded the product **PB**? What if we wanted to quantify **time T**? We would simply rewrite the equation as

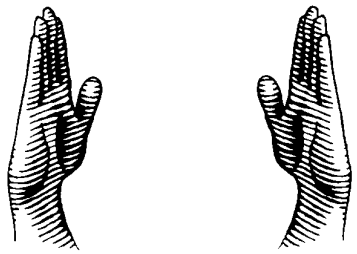
$$\mathbf{T = \frac{PB \times PC}{E}\ ;}$$

so that now we see that effort **E** is inversely proportional to the amount of time spent. In other words, if one works harder, one spends less time doing the work. Hmm. ***Important note:*** *if you are not comfortable with this type of algebraic manipulation, please ask for guidance about this from a tutor, mentor or math teacher. You* ***must*** *be able to do this quickly and effortlessly, and you certainly can with a bit of practice. A useful website: www.chem.tamu.edu/class/fyp/mathrev/mr-algeb.html.*

The Importance of Constants

Oftentimes, an equation you use will involve a constant, or a certain value with specific dimensional units which must be factored in to your calculations so that things work out reasonably. It is important to determine whether you must memorize these constants. Some instructors give you the constants as part of an exam or problem set. A constant is the special ingredient to the formula which makes the whole thing "set" right. Do not leave your secret ingredient out of the recipe.

Dimensional Analysis – Size Matters



Dimensional analysis is a system of **assigning units** to numerical values based upon whether the number denotes a distance, a mass, a quantity of energy, an amount of light, a period of time, etc. The kicker in word problems is to be able to translate the

units you are given into the units you must use **in the standard equation**, and also into the units you are asked for in your answer to the problem. The key words will often be "express your answer as *buckets of water per match*," or something like that. If your standard formula only enables you to solve for *drops of water per barn fire*, then you will have to do some **converting** of the values to have the answer ***make sense*** in terms of magnitude.

Below is a quick, five-step guide for doing dimensional analysis. It is taken from Bob Jacobs' Chemistry Coach website. The page is http://www.chemistrycoach.com/use.htm. You will also find some plain language (i.e., non-scientific) practice problems at www.chemistrycoach.com/dimensiional_analysis_problem_set.htm.

A Method for Dimensional Analysis

Problem: Convert 5 grams (gm) to milligrams (mg).

Before you begin, you should already have an idea of which is going to be the larger number. You can fit many, many milligrams into just one gram. So, to express five grams as a milligram amount is going to take up more space on your paper in terms of digits. Can you see that?

Step 1. Write the term to be converted, both the amount and the unit:

5 gm

Step 2. Write the conversion formula/e to use:

1,000 mg = 1 gm

Step 3. Turn the conversion formula into a fraction as follows –

- if the units of the Step 1 figure are in the numerator, then that unit must be in the denominator of your fraction.

- if the units of the Step 1 figure are in the denominator, then that unit must be in the numerator of this step.

$$\frac{\textbf{1,000mg}}{\textbf{1 gm}}$$

Note that these values are equivalent, so this fraction is equal to 1.

Step 4. Multiply the Step 1 term by the Step 3 fraction. Again, you are multiplying by one, so you are not changing the *value*, but only **the units** of the term.

$$5 \text{ gm} \times \frac{1{,}000 \text{mg}}{1 \text{ gm}}$$

Step 5. Cancel units and perform calculation, maintaining correct number of significant figures.

$$5 \text{ ~~gm~~} \times \frac{1{,}000 \text{mg}}{1 \text{ ~~gm~~}} = 5{,}000 \text{ mg} = 5 \times 10^3 \text{ mg}$$

Numerical Values Ride Along

Now that you have written everything down and have found your formula with which to solve the problem, you "plug and chug," or do the calculations with the pertinent values according to the dimensional analysis method. Please notice that ***the numbers are just along for the ride*** by the time you have figured out the layout of the problem. Were you to change the names of the people in a book or a play, you would still have the self-same plot. "A rose by any other name would smell as sweet." Values are just things that make your exercise possible. Allow them to be "carried along" (see Garrigan, G. Strategies for Solving Word Problems in Science, 1997. From: http://wwww.vccaedu.org/inquiry/inquiry-fall97/i12-garr.html.)

Final Analysis: Is the Answer Reasonable?

When all the calculating is done, units canceled and everything accounted for, take a good look at your answer and determine the following honestly:

1. that the **units** go with the kind of term the question seeks – for example, units of work done are given in "ergs"; did you end up with an answer in the same?
2. that the quantity, or numerical amount you have come up with is sensible – for example, you had a

problem in which you are calculating a distance someone could walk in an afternoon. Did he end up on the moon, or in the next town?

3. that your sign is correct; i.e., positive or negative. Sometimes the sign indicates direction, or a gain or loss. Be sure that your sign indicates whether you have gone up or down if this is applicable.

Review

Okay, let's review the steps for solving word problems.
Number One, for the rest of your life, always remember and *never forget* is:

1. **WRITE AS YOU READ!!!**
2. **Determine the formula and include necessary constant/s.**
3. **Use dimensional analysis to convert terms.**
4. **Allow numbers to be "carried along" in the calculation.**
5. **Analyze your answer for reasonableness.**
6. **Enjoy your success ;-)**