

MISCELLANEOUS IDENTITIES

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} \quad \cot \alpha = \frac{\cos \alpha}{\sin \alpha} \quad \sin \alpha = \frac{1}{\csc \alpha} \quad \cos \alpha = \frac{1}{\sec \alpha} \quad \sec \alpha = \frac{1}{\cos \alpha} \quad \csc \alpha = \frac{1}{\sin \alpha}$$

$$\sin(-\alpha) = -\sin \alpha \quad \cos(-\alpha) = \cos \alpha \quad \tan(-\alpha) = -\tan \alpha \quad \cot(-\alpha) = -\cot \alpha$$

$$\sec(-\alpha) = \sec \alpha \quad \csc(-\alpha) = -\csc \alpha$$

COFUNCTION RELATIONS

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha \quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha \quad \tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha \quad \cot\left(\frac{\pi}{2} - \alpha\right) = \tan \alpha$$

$$\sec\left(\frac{\pi}{2} - \alpha\right) = \csc \alpha \quad \csc\left(\frac{\pi}{2} - \alpha\right) = \sec \alpha$$

PHYTHAGOREAN RELATIONS

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad \sec^2 \alpha = 1 + \tan^2 \alpha \quad \csc^2 \alpha = 1 + \cot^2 \alpha$$

ANGLE SUM AND DIFFERENCE RELATIONS

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

DOUBLE ANGLE RELATIONS

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha \quad \cos 2\alpha = 2 \cos^2 \alpha - 1 = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

HALF ANGLE RELATIONS

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

PRODUCT RELATIONS

$$2 \sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad 2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

$$2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta) \quad 2 \cos \alpha \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

SUM AND DIFFERENCE RELATIONS

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

TRIGONOMETRIC FUNCTIONS

EXACT VALUES FOR TRIGONOMETRIC FUNCTIONS OF VARIOUS ANGLES

	Angle A in degrees	Angle A in radians	$\sin A$	$\cos A$	$\tan A$	$\cot A$	$\sec A$	$\csc A$
-2π	0°	0	0	1	0	∞	1	∞
	15°	$\pi/12$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$2-\sqrt{3}$	$2+\sqrt{3}$	$\sqrt{6}-\sqrt{2}$	$\sqrt{6}+\sqrt{2}$
$-\frac{11\pi}{6}$	30°	$\pi/6$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	2
$-\frac{7\pi}{4}$	45°	$\pi/4$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$-\frac{5\pi}{3}$	60°	$\pi/3$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	2	$\frac{2}{3}\sqrt{3}$
	75°	$5\pi/12$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$2+\sqrt{3}$	$2-\sqrt{3}$	$\sqrt{6}+\sqrt{2}$	$\sqrt{6}-\sqrt{2}$
$-\frac{3\pi}{2}$	90°	$\pi/2$	1	0	$\pm\infty$	0	$\pm\infty$	1
	105°	$7\pi/12$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-(2+\sqrt{3})$	$-(2-\sqrt{3})$	$-(\sqrt{6}+\sqrt{2})$	$\sqrt{6}-\sqrt{2}$
$-\frac{4\pi}{3}$	120°	$2\pi/3$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	-2	$\frac{2}{3}\sqrt{3}$
$-\frac{5\pi}{4}$	135°	$3\pi/4$	$\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$-\frac{7\pi}{6}$	150°	$5\pi/6$	$\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	$-\sqrt{3}$	$-\frac{2}{3}\sqrt{3}$	2
	165°	$11\pi/12$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$-(2-\sqrt{3})$	$-(2+\sqrt{3})$	$-(\sqrt{6}-\sqrt{2})$	$\sqrt{6}+\sqrt{2}$
$-\pi$	180°	π	0	-1	0	$\mp\infty$	-1	$\pm\infty$
	195°	$13\pi/12$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$2-\sqrt{3}$	$2+\sqrt{3}$	$-(\sqrt{6}-\sqrt{2})$	$-(\sqrt{6}+\sqrt{2})$
$-\frac{5\pi}{6}$	210°	$7\pi/6$	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$	$-\frac{2}{3}\sqrt{3}$	-2
$-\frac{3\pi}{4}$	225°	$5\pi/4$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
$-\frac{2\pi}{3}$	240°	$4\pi/3$	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$	-2	$-\frac{2}{3}\sqrt{3}$
	255°	$17\pi/12$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$2+\sqrt{3}$	$2-\sqrt{3}$	$-(\sqrt{6}+\sqrt{2})$	$-(\sqrt{6}-\sqrt{2})$
$-\frac{\pi}{2}$	270°	$3\pi/2$	-1	0	$\pm\infty$	0	$\mp\infty$	-1
	285°	$19\pi/12$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-(2+\sqrt{3})$	$-(2-\sqrt{3})$	$\sqrt{6}+\sqrt{2}$	$-(\sqrt{6}-\sqrt{2})$
$-\frac{\pi}{3}$	300°	$5\pi/3$	$-\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	2	$-\frac{2}{3}\sqrt{3}$
$-\frac{\pi}{4}$	315°	$7\pi/4$	$-\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
$-\frac{\pi}{6}$	330°	$11\pi/6$	$-\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{3}\sqrt{3}$	$-\sqrt{3}$	$\frac{2}{3}\sqrt{3}$	-2
	345°	$23\pi/12$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$-(2-\sqrt{3})$	$-(2+\sqrt{3})$	$\sqrt{6}-\sqrt{2}$	$-(\sqrt{6}+\sqrt{2})$
0	360°	2π	0	1	0	$\mp\infty$	1	$\mp\infty$

For tables involving other angles see pages 206-211 and 212-215.