

TRANSLATING ENGLISH PHRASES INTO ALGEBRAIC FORM

	<u>English Phrase</u>	<u>Algebraic Form</u>
Addition	The sum of a number and three.	$x + 3$ or $3 + x$
	Three more than a number.	
	A number increased by three.	
Subtraction	Three diminished by a number.	$3 - x$
	Three less a number.	
	Three decreased by a number.	
	The difference of three and a number.	
	Three less than a number.	$x - 3$
	Three subtracted from a number.	
Multiplication	Three times a number.	$(3)(x)$, $3 \cdot x$, $3(x)$ or just $3x$ ("3x" is the preferred way)
	The product three and a number.	
	A number is tripled.	
Division	A number divided by three.	$x \div 3$ or $\frac{x}{3}$
	The quotient of a number and three.	
		$(\frac{x}{3}$ is the preferred way)
	Three divided by a number	
	$\frac{3}{x}$	
The quotient of three and a number		

* *Note:* The phrases **less than** and **subtracted from** reverse the order when translating from English into math symbols.

Use x as the variable.

<u>English Phrase</u>	<u>Algebraic Form</u>
1. Some number subtracted from 10.	1. _____
2. The quotient of some number and 6.	2. _____
3. The difference of some number and 5.	3. _____
4. Some number decreased by 5.	4. _____
5. The product of 8 and the sum of some number and 9.	5. _____
6. 4 less than 5 times some number.	6. _____
7. 8 subtracted from 6 times some number.	7. _____
8. 11 decreased by 5 times some number.	8. _____
9. The product of some number and three more than the number.	9. _____
10. The product of 4 and the difference of some number and 6 divided by the sum of the same number and 2.	10. _____

ANSWER KEY

- | | |
|------------------|--------------------------------|
| 1. $10 - x$ | 6. $5x - 4$ |
| 2. $\frac{x}{6}$ | 7. $6x - 8$ |
| 3. $x - 5$ | 8. $11 - 5x$ |
| 4. $x - 5$ | 9. $x(x + 3)$ |
| 5. $8(x + 9)$ | 10. $\frac{4(x - 6)}{(x + 2)}$ |

TRANSLATING ALGEBRAIC EXPRESSIONS

Translate the following phrases into algebraic expressions (unless otherwise instructed, use "x" as the variable):

1. Seven less a number _____
2. Three more than a number _____
3. The sum of a number and 11 _____
4. Twice y added to x _____
5. Twelve less than a number _____
6. The product of 7 and a number _____
7. The quotient of 15 and a number _____
8. Five less than 6 times a number _____
9. The sum of 9 and x subtracted from y _____
10. 7 times the sum of x and y _____
11. The sum of x squared and y cubed _____
12. Thirty-seven decreased by the difference of 18 and a number _____
13. The product of the base (b) and height (h) divided by two _____
14. Negative 3 times the sum of x and twice y _____
15. Four times a number less the sum of 5 times a number and 8 _____
16. x squared added to 5 times a number _____
17. The sum of x and 13 subtracted from x squared _____
18. Twice the length (L) added to twice the width (W) _____
19. The sum of 5 and the quotient of 7 and x _____
20. The difference of x and y is multiplied by z _____
21. The product of the length (L), the width (W) and the height (H) _____
22. The sum of x squared and twice x , divided by the difference of x squared and x _____

Make a list of words you find in word problems that mean addition, subtraction, multiplication, division, or equal.

+	-	x	÷	=

Words that reverse the order of placement:

ANSWER KEY

- | | |
|----------------------------------|---|
| 1. $7 - x$ | 12. $37 - (18 - x)$ |
| 2. $x + 3$ or $3 + x$ | |
| 3. $x + 11$ | 13. $\frac{bh}{2}$ |
| 4. $x + 2y$ or $2y + x$ | 14. $-3(x + 2y)$ |
| 5. $x - 12$ | 15. $4x - (5x + 8)$ |
| 6. $7 \cdot x$ or $7x$ | 16. $5x + x^2$ or $x^2 + 5x$ |
| 7. $15 \div x$ or $\frac{15}{x}$ | 17. $x^2 - (x + 13)$ |
| 8. $6x - 5$ | 18. $2L + 2W$ |
| 9. $y - (9 + x)$ | 19. $5 + 7 \div x$ or $5 + \frac{7}{x}$ |
| 10. $7(x + y)$ | 20. $z(x - y)$ |
| 11. $x^2 + y^3$ | 21. $L \cdot W \cdot H$ or LWH |
| | 22. $(x^2 + 2x) \div (x^2 - x)$ or $\frac{x^2 + 2x}{x^2 - x}$ |

Translating Sentences into Equations and Solving

Example 1: "A number decreased by six equals fifteen".

Let $x =$ the number equation: $x - 6 = 15$

Solve:
$$\begin{array}{r} x - 6 = 15 \\ + 6 \quad + 6 \\ \hline x = 21 \end{array}$$

$x = 21$

Example 2: The product of two and a number is ten.

Let $x =$ the number equation: $2x = 10$

Solve:
$$\frac{2x}{2} = \frac{10}{2}$$

$x = 5$

Example 3: The sum of three times a number and six equals four. Find the number.

Let $x =$ the number equation: $3x + 6 = 4$

Solve:
$$\begin{array}{r} 3x + 6 = 4 \\ - 6 \quad - 6 \\ \hline 3x = -2 \\ \frac{3x}{3} = \frac{-2}{3} \end{array}$$

$x = -\frac{2}{3}$

Example 4: Three more than four times a number is ten. Find the number.

Let $x =$ the number equation: $4x + 3 = 10$

Solve:
$$\begin{array}{r} 4x + 3 = 10 \\ - 3 \quad - 3 \\ \hline 4x = 7 \\ \frac{4x}{4} = \frac{7}{4} \end{array}$$

$x = \frac{7}{4}$

Example 5: The sum of two consecutive integers is 27. Find the integers.

Let $x =$ First consecutive integer equation: $x + (x + 1) = 27$

$x + 1 =$ Second consecutive integer Solve: $x + x + 1 = 27$

$$\begin{array}{r} 2x + 1 = 27 \\ - 1 \quad - 1 \\ \hline 2x = 26 \\ \frac{2x}{2} = \frac{26}{2} \end{array}$$

$13, 14$

Example 6: The sum of two consecutive odd integers is 40. Find the integers.

Let $x =$ First consecutive ODD integer equation: $x + (x + 2) = 40$

$x + 2 =$ Second consecutive ODD integer Solve: $x + x + 2 = 40$

$$\begin{array}{r} 2x + 2 = 40 \\ - 2 \quad - 2 \\ \hline 2x = 38 \\ \frac{2x}{2} = \frac{38}{2} \end{array}$$

$19, 21$

Example 7: The sum of three consecutive even integers is -24. Find the integers.

Let $x =$ First consecutive EVEN integer equation: $x + (x + 2) + (x + 4) = -24$

$x + 2 =$ Second consecutive EVEN integer Solve: $x + x + 2 + x + 4 = -24$

$x + 4 =$ Third consecutive EVEN integer

$$\begin{array}{r} 3x + 6 = -24 \\ - 6 \quad - 6 \\ \hline 3x = -30 \\ \frac{3x}{3} = \frac{-30}{3} \end{array}$$

$-10, -12, -14$

1. The sum of a number and seven is twelve. Find the number.

Let $x =$ _____ equation;

Solve:

2. A number decreased by seven is five. Find the number.

Let $x =$ _____ equation;

Solve:

3. Five more than a number is three. Find the number.

Let $x =$ _____ equation;

Solve:

4. Six times a number is fourteen. Find the number.

Let $x =$ _____ equation;

Solve:

5. The total of twenty and a number is five. Find the number.

Let $x =$ _____ equation;

Solve:

6. The sum of three times a number and four is eight. Find the number.

Let $x =$ _____ equation;

Solve:

7. Seven less than four times a number is nine. Find the number.

Let $x =$ _____ equation;

Solve:

8. The total of a number divided by four and nine is two. Find the number.

Let $x =$ _____ equation;

Solve:

9. Six less than the quotient of a number and four is equal to negative two. Find the number.

Let $x =$ _____ equation;

Solve:

10. The difference between seven and twice a number is thirteen. Find the number.

Let $x =$ _____ equation;

Solve:

11. The total of ten times a number and seven is twenty-seven. Find the number.

Let $x =$ _____ equation;

Solve:

12. The sum of three fifths of a number and eight is two. Find the number.

Let $x =$ _____ equation;

Solve:

13. The sum of two consecutive integers is 47. Find the integers.

Let $x =$ _____ equation;
_____ = _____ Solve:

14. The sum of two consecutive odd integers is 172. Find the integers.

Let $x =$ _____ equation;
_____ = _____ Solve:

15. The quotient of a number and negative three is 21. Find the number.

Let $x =$ _____ equation;
Solve:

16. If a number is decreased by 18 the result is -2. Find the number.

Let $x =$ _____ equation;
Solve:

17. The sum of three consecutive integers is 57. Find the three integers.

Let $x =$ _____ equation;
_____ = _____ Solve:
_____ = _____

Answer Key

Translating Sentences and Solving

1. 5

2. 12

3. -2

4. $\frac{7}{3}$

5. -15

6. $\frac{4}{3}$

7. 4

8. -28

9. 16

10. -3

11. 2

12. -10

13. 23, 24

14. 85, 87

15. -63

16. 16

17. 18, 19, 20

M01C
WORD PROBLEMS

MAT 0024

Translate the following sentences and solve the equations:

1. If 2 is subtracted from five times a certain number the result is 28.

2. Six added to a certain number is 29. What is the number?

3. A certain number subtracted from 23 is 14. Find the number.

4. If 4 is added to five times a certain number, the result is 39. Find the number.

5. If 2 is subtracted from seven times a certain number, the result is -30. Find the number.

6. Seventeen is two larger than three times a certain number. Find the number.

7. Twenty is 22 less than six times a certain number. Find the number.

8. If eight times a certain number is subtracted from 27, the result is 3. Find the number.

9. The sum of a number plus three times the number is 84. What is the number?

10. Find two consecutive even integers whose sum is 66.

11. Find two consecutive odd integers whose sum is 92.

12. Find three consecutive integers whose sum is 102.

13. Five times a certain number minus 3 is equal to seven times the same number minus 13. Find the number.

14. Two less than three times a certain number is the same as 13 more than eight times the number. What is the number?

15. The sum of a number and three times the number is equal to 18 more than twice the number. Find the number.

16. Find two consecutive integers such that the first plus five times the second equals 107.

17. Find four consecutive integers such that the sum of the first three equals the fourth number.

Answer Key

1. $5x - 2 = 28$ $x = 6$
2. $6 + x = 29$ $x = 23$
3. $23 - x = 14$ $x = 9$
4. $4 + 5x = 39$ $x = 7$
5. $7x - 2 = -30$ $x = -4$
6. $17 = 2 + 3x$ $x = 5$
7. $20 = 6x - 22$ $x = 7$
8. $27 - 8x = 3$ $x = 3$
9. $x + 3x = 84$ $x = 21$
10. $x + (x + 2) = 66$ 32, 34
11. $x + (x + 2) = 92$ 45, 47
12. $x + (x + 1) + (x + 2) = 102$ 33, 34, 35
13. $5x - 3 = 7x - 13$ $x = 5$
14. $3x - 2 = 13 + 8x$ $x = -3$
15. $x + 3x = 18 + 2x$ $x = 9$
16. $x + 5(x + 1) = 107$ 17, 18
17. $x + (x + 1) + (x + 2) = x + 3$ 0, 1, 2, 3

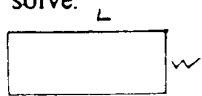
The perimeter of any figure is the total distance around that figure. For a rectangle in particular, the perimeter consists of two lengths and two widths. $P = 2l + 2w$

EXAMPLE: The length of a rectangle is 8 feet more than twice the width. The perimeter is 106 feet. Find the dimensions.

SOLUTION: Step 1 - Write down what you will call the unknown.

$$\begin{aligned} \text{Let } x &= \text{the width} \\ 2x + 8 &= \text{the length} \end{aligned}$$

Step 2 - Draw the shape, write down the formula, replace the variables in the formula, and solve.



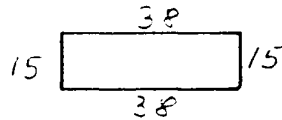
$$\begin{aligned} P &= 2L + 2W \\ 106 &= 2(2x + 8) + 2x \\ 106 &= 4x + 16 + 2x \\ 106 &= 6x + 16 \\ \underline{-16} & \qquad \qquad \underline{-16} \\ 90 &= 6x \end{aligned}$$

$$\begin{aligned} \frac{90}{6} &= \frac{6x}{6} & x &= 15 \\ & & 2x + 8 &= 38 \end{aligned}$$

Step 3 - Answer the question.

The width is 15 feet and the length is 38 feet

Step 4 - Check your answers.



$$15 + 38 + 15 + 38 = 106 \quad \checkmark$$

$$\begin{aligned} \text{or } 2(15) + 2(38) &= 106 \\ 30 + 76 &= 106 \quad \checkmark \end{aligned}$$

PROBLEMS:

1. The length of a rectangle is 5 feet more than the width. The perimeter is 130 feet. Find the dimensions.

2. The length of a rectangle is twice the width. The perimeter is 24 feet. Find the dimensions.

3. The length of a rectangle is 4 feet more than twice the width. The perimeter is 80 feet. Find the dimensions.

4. The perimeter of a rectangular field is 300 meters. If the length is twice the width, what are the dimensions of the field?

5. The length of a rectangle is 4 times the width. The perimeter is 120 meters. Find the length and width.

6. The length of a rectangle is 5 feet less than twice the width; the perimeter is 50 feet. Find the length and width.

7. The width of a rectangle is 50 feet less than the length. If the perimeter is 400 feet, find the length and width.

8. One side of a triangle is 10 centimeters longer than the shortest side. A third side is 20 centimeters larger than the shortest side. The perimeter of the triangle is 120 centimeters. Find the length of the shortest side.

9. The length of a rectangle is three feet less than five times the width. Their perimeter is ten times the width. Find the dimensions and the perimeter.
10. The perimeter of a triangle is 43 meters. The length of the second side is five meters more than the first, and the third is three more than the second. Find the length of each.

ANSWER KEY

- | | | |
|-----|--|---|
| 1. | Let x = the width
$x + 5$ = the length | $130 = 2(x + 5) + 2x$
The width is 30 feet, the length is 35 feet. |
| 2. | Let x = the width
$2x$ = the length | $24 = 2(2x) + 2x$
The width is 4 feet, the length is 8 feet. |
| 3. | Let x = the width
$2x + 4$ = the length | $80 = 2(2x + 4) + 2x$
The width is 12 feet, the length is 28 feet. |
| 4. | Let x = the width
$2x$ = the length | $300 = 2(2x) + 2x$
The width is 50 meters, the length is 100. |
| 5. | Let x = the width
$4x$ = the length | $120 = 2(4x) + 2x$
The width is 12 meters, the length is 48 meters. |
| 6. | Let x = the width
$2x - 5$ = the length | $50 = 2(2x - 5) + 2x$
The width is 10 feet, the length is 15 feet. |
| 7. | Let x = the length
$x - 50$ = the width | $400 = 2x + 2(x - 50)$
The length is 125 feet, the width is 75 feet. |
| 8. | Let x = the shortest side
$x + 10$ = the second side
$x + 20$ = the third side | $120 = x + x + 10 + x + 20$
The shortest side is 30 cm. |
| 9. | Let x = the width
$5x - 3$ = the length
$10x$ = the perimeter | $10x = 2(5x - 3) + 2x$
The width is 3 feet, the length is 12 feet, and the perimeter is 30 feet. |
| 10. | Let x = the first side
$x + 5$ = the second side
$x + 8$ = the third side | $43 = x + x + 5 + x + 8$
The first side is 10 meters, the second side is 15 meters, and the third side is 18 meters. |

In a coin problem (and other types as well), making a chart may be helpful. Fill in the first two columns with variables and given information. Multiplying the first column times the second column gives the third column. The equation may be found by adding down the last column and setting it equal to the grand total.

SAMPLE CHART:

	Number of Coins	× Value of Each	= Total Value
Type 1			
Type 2			
Type 3			
			GRAND TOTAL

EXAMPLE: A box contains nickels, dimes and quarters worth \$2.10. There are twice as many dimes as quarters, and the number of nickels is two less than the number of dimes. How many of each are there?

SOLUTION: The problem says "twice as many dimes as quarters" so jump on that last word in the phrase "quarters" and

Let x = number of quarters

$2x$ = number of dimes

$2x - 2$ = number of nickels

Obviously, the value of quarters is 25

the value of dimes is 10

the value of nickels is 5

The grand total value is 210 cents or \$2.10.

	Number of Coins	× Value of Each	= Total Value
Quarters	x	25	$25(x)$
Dimes	$2x$	10	$10(2x)$
Nickels	$2x - 2$	5	$5(2x - 2)$
			210

The equation (from the last column) is $25(x) + 10(2x) + 5(2x - 2) = 210$
 $25x + 20x + 10x - 10 = 210$

Solve:

$$55x = 220$$

$$x = 4 \text{ quarters}$$

$$2x = 8 \text{ dimes}$$

$$2x - 2 = 6 \text{ nickels}$$

Check:

\$1.00	Quarters
.80	Dimes
<u>.30</u>	Nickels
\$2.10	

1. A certain number of quarters and four times as many pennies are worth \$1.45. How many of each coin are there?

SOLUTION: Let x = number of _____
 $4x$ = number of _____

	Number of Coins	× Value of Each	= Total Value
Quarters			
Pennies			

Write the equation: _____

Solve the Equation:

Answer the question: x = _____ Quarters
 $4x$ = _____ Pennies

Check:

2. A certain number of dimes and four times as many pennies are worth \$0.98. How many of each coin are there?

SOLUTION: Let x = number of _____
 $4x$ = number of _____

	Number of Coins	× Value of Each	= Total Value
Dimes			
Pennies			

Write the equation _____

Solve the Equation

Answer the question: _____

Check:

3. A certain number of quarters and three times as many dimes are worth \$1.65. How many of each are there?

SOLUTION: Let x = _____
 $3x$ = _____

	Number of Coins	× Value of Each	= Total Value
Quarters			
Dimes			

Write the equation: _____

Solve the Equation: _____

Answer the question: _____

Check: _____

4. A certain number of quarters, four times as many pennies as quarters, and 6 more dimes than pennies are worth \$3.36. How many of each coin are there?

SOLUTION: Let x = number of _____
 _____ = number of pennies
 _____ = number of dimes

	Number of Coins	× Value of Each	= Total Value
Quarters	x		$25(x)$
Pennies	$4x$		$1(4x)$
Dimes	$4x + 6$		$10(4x + 6)$
			336

Write the equation: _____

Solve the Equation: _____

Answer the question: _____

Check: _____

5. A box contains 30 coins, in nickels and dimes, worth \$2.40. How many of each coin are there?

SOLUTION: Let x = number of nickels
 $30 - x$ = number of dimes

	Number of Coins	× Value of Each	= Total Value
Nickels			
Dimes			

Write the equation: _____

Solve the Equation: _____

Answer the question: _____

Check: _____

6. A box contains 20 coins, in quarters and dimes, worth \$2.90. How many of each coin are there?

SOLUTION: Let x = number of _____
 _____ = _____

	Number of Coins	× Value of Each	= Total Value
Quarters			
Dimes			

Write the equation: _____

Solve the Equation: _____

Answer the question: _____

Check: _____

7. A box contains 20 coins, in quarters and dimes, worth \$3.80. How many of each coin are there?

SOLUTION: Let x = number of _____
 _____ = _____

Make your own chart:

Write and solve the equation:

Answer the question: _____

Check:

8. A box contains \$6.60 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each are there?

SOLUTION: Let x = number of _____
 _____ = _____
 _____ = number of dimes (NOT $4 - 3x$)

Number of Coins	× Value of Each	= Total Value

Write the equation: _____

Solve the Equation:

Answer the question: _____

Check:

9. A certain number of pennies, four times as many dimes as pennies, and a number of quarters which is 16 less than twice the number of dimes, are worth \$24.92. How many of each are there?

SOLUTION:

10. A box contains \$8.00 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each coin are there?

SOLUTION:

11. A sum of money consists of nickels, dimes, and quarters amounting to \$1.90. If there are twice as many nickels as quarters and three less dimes than nickels, how many of each are there?

SOLUTION:

ANSWERS: 1) 5 quarters and 20 pennies; 2) 7 dimes and 28 pennies; 3) 3 quarters, 9 dimes; 4) 4 quarters, 22 dimes and 16 pennies; 5) 18 dimes and 12 nickels; 6) 6 quarters and 14 dimes; 7) 12 quarters and 8 dimes; 8) 10 quarters, 26 dimes, and 30 nickels; 9) 80 quarters, 48 dimes, and 12 pennies; 10) 12 quarters, 32 dimes and 36 nickels; 11) 4 quarters, 5 dimes and 8 nickels