SUMMARY OF LAWS OF EXPONENTS

	GENERALIZATION
1. When you multiply (with same base number), you add the exponents.	$x^m \cdot x^n = x^{m+n}$
2. When you divide (with same base number), you subtract the exponents.	$\frac{x^m}{x^n} = x^{m-n}$
3. When you raise a power to a power, you multiply the exponents.	$(x^m)^n = x^{mn}$
4. When a product or quotient is raised to a power, you raise each factor to the power.	$(xy)^n = x^n y^n$ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
5. Any non-zero number raised to the zero power is 1.	$x^0 = 1$
6. A number raised to a negative power is 1 divided by that number raised to the positive power.	$x^{-n} = \frac{1}{x^n}$
7. One (1) divided by a number raised to a negative power is that number raised to the positive power.	$\frac{1}{x^{-n}} = x^n$
8. A fraction raised to a negative power is the reciprocal of the fraction raised to the positive power.	$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

Be sure all order of operation are observed. The best way to learn and remember these rules is to practice them until they become familiar and the problems become easy.

Try problems 27 - 81 Perform the indicated operation. Leave answers with **positive** exponents.

27. $4^7 \cdot 4^{-3}$

28. $3^{-7} \cdot 3^3$

29. $x^{-7} \cdot x^5$

30. $a^5 b^4 a^{-2} b^2$

31. $x^{-3} y^2 x^{-2} y^5$

32. $x^{-3} y^{-2} x^{-5} y^{-4}$

33. $(x^{-2}s^{2})^{3}$

34. $(a^{3}b^{-5})^{4}$

35. $(z^{-2}w^{3})^{-4}$

36. $(2x^{3})^{-2}$

37. $(4x^{2})^{-3}$

38. $(-4x^{-3})^{-4}$

39. $(-2y^{-5})^{2}$

40. $(2x^{-3}y^{-2})^{3}$

41. $(5a^{-2}b^{-5})^{2}$

42. $(3x^{3}y^{-4})^{-4}$

43. $(2w^{3}z^{-2})^{-2}$

44. $\left(\frac{x^{3}}{y^{5}}\right)^{3}$

45. $\left(\frac{w^{3}}{z^{-6}}\right)^{3}$

46. $\left(\frac{a^{5}}{b^{3}}\right)^{-5}$

47. $\left(\frac{x^{4}}{z^{6}}\right)^{-5}$

48. $\frac{x^{2}\cdot x^{7}}{x^{12}\cdot x^{6}}$

49. $\frac{w^{3}\cdot (w^{4})^{-2}}{w^{2}\cdot w^{2}}$

50. $(3a^{5}b^{-2}c^{3})^{-2}$

51. $\frac{(a^{5})^{-2}(b^{-2})^{-2}}{a^{8}b^{4}}$

52. $(-3x^{2}y^{-2}c^{-3})^{-2}$

53. $\frac{(y^{3})^{4}}{(y^{5})^{2}}$

54. $\frac{(a^{-3})^{3}}{(a^{3})^{2}}$

55. $\frac{(z^{-2})^{4}}{(z^{5})^{2}}$

56. $\frac{(a^{-2})^{-3}}{(a^{-4})^{2}}$

57. $\frac{(p^{-4})^{-2}}{(p^{-5})^{3}}$

58. $\frac{(2y^{2})^{4}}{(y^{4})^{4}}$

59. $\frac{(3q^{3})^{3}}{(q^{2})^{4}}$

60. $\frac{(x^{2})^{5}}{x^{2}x^{4}}$

61. $\frac{(y^{4})^{3}}{y^{3}y^{2}}$

62. $\frac{(3x^4)^0}{(2x^2)^2}$

63. $\frac{(5c^3)^2}{(8c^6)^0}$

64. $\frac{(7^0g^4)^3}{(2g^2)^4}$

65. $\frac{(4a^4)^3}{(5^0a^2)^4}$

66. $\frac{(x^{-2})^4}{x^3x^4}$

67. $\frac{(z^{-1})^5}{z^5z^2}$

68. $\frac{(q^2)^3(q^4)^2}{(q^3)^5}$

69. $\frac{(p^4)^2(p^5)^3}{(p^2)^7}$

70. $\frac{(4a^{-4})^3(2a^2)^{-3}}{(2a^{-2})^3}$

71. $\frac{(3b^5)^{-3}(2b^3)^{-3}}{(3b^4)^{-2}}$

72. $\frac{(2x^{-3})^3}{x^2x}$

73. $\frac{(2y^{-4})^4}{y^5y^2}$

74. $\frac{(-3a^{-3})^4}{(a^{-3})^5}$

75. $\frac{6^{-1}y^2z^5}{6^2y^{-1}z^{-2}} =$

76. $\frac{6y^2z^5}{6^2yz} =$

77. $\frac{x^{45}y^{100}}{x^{15}y^{10}} =$

78. $\left(\frac{r^{-2}s^{-3}}{r^{-4}s^{-3}}\right)^{-3}$

79. $\left(\frac{-3x^2y^2}{xyz}\right)^{-2}$

80. $\frac{(2x^{-3}y^2)^4(3xy^{-3})^{-2}}{(3x^{-4}y^{-2})^{-4}}$

81. $\frac{(4a^{-3}b^{-2})^{-2}(6a^3b^{-5})^3}{(2a^{-2}b^{-2})^{-4}}$

** Check your answers against the key. **

ANSWER KEY

1. $\frac{1}{x^3}$ 2. $\frac{1}{25}$ 3. $\frac{1}{8}$ 4. $\frac{1}{32}$ 5. $\frac{1}{y^6}$ 6. $\frac{1}{m}$

$\frac{1}{36}$ 8. $\frac{1}{4}$ 9. y^3 10. m^7 11. 25 12. -8 13. 9

14. 16 15. $\frac{4}{25}$ 16. $\frac{25}{9}$ 17. $\frac{16}{25}$ 18. $\frac{3}{5}$ 19. $\frac{7}{3}$ 20. $\frac{8}{27}$

21. $\frac{1}{5m}$ 22. $\frac{5}{m}$ 23. $\frac{1}{2z}$ 24. $\frac{8}{b^2}$ 25. $\frac{1}{8c}$ 26. $\frac{3}{t}$ 27. 256

28. $\frac{1}{81}$ 29. $\frac{1}{x^2}$ 30. a^3b^6 31. $\frac{y^7}{x^5}$ 32. $\frac{1}{x^8y^6}$ 33. $\frac{s^6}{x^6}$ 34. $\frac{a^{12}}{b^{20}}$

35. $\frac{z^8}{w^{12}}$ 36. $\frac{1}{4x^6}$ 37. $\frac{1}{64x^6}$ 38. $\frac{x^{12}}{256}$ 39. $\frac{4}{y^{10}}$ 40. $\frac{8}{x^9y^6}$ 41. $\frac{25}{a^4b^{10}}$

42. $\frac{y^{16}}{81x^{12}}$ 43. $\frac{z^4}{4w^6}$ 44. $\frac{x^9}{y^{15}}$ 45. w^9z^{18} 46. $\frac{b^{15}}{a^{25}}$ 47. $\frac{z^{30}}{x^{20}}$ 48. $\frac{1}{x^9}$

49. $\frac{1}{w^9}$ 50. $\frac{b^4}{9a^{10}c^6}$ 51. $\frac{1}{a^{18}}$ 52. $\frac{y^4c^6}{9x^4}$ 53. y^2 54. $\frac{1}{a^{15}}$

55. $\frac{1}{z^{18}}$ 56. a^{14} 57. p^{23} 58. $\frac{16}{y^8}$ 59. 27q 60. x^4 61. y^7

62. $\frac{1}{4x^4}$ 63. $25c^6$ 64. $\frac{q^4}{16}$ 65. $64a^4$ 66. $\frac{1}{x^{15}}$ 67. $\frac{1}{z^{12}}$ 68. $\frac{1}{q}$

69. p^9 70. $\frac{1}{a^{12}}$ 71. $\frac{1}{24b^{16}}$ 72. $\frac{8}{x^{12}}$ 73. $\frac{16}{y^{23}}$ 74. $81a^3$ 75. $\frac{y^3z^7}{216}$

76. $\frac{yz^4}{6}$ 77. $x^{30}y^{90}$ 78. $\frac{1}{r^6}$ 79. $\frac{z^2}{9x^2y^2}$ 80. $\frac{144y^6}{x^{30}}$ 81. $\frac{216a^7}{b^{19}}$

Introduction to Negative Exponents

From your work with positive exponents you know that the expression $\frac{x^5}{x^3}$ simplifies to x^2, because $\frac{x^5}{x^3} = \frac{xxxxx}{xxx} = x^2$. In section 3.1 you were introduced to the **Quotient Rule**, a short cut to such problems as the one shown above. The result would have been the same if we had used this rule (**subtracted exponents** of the common base).

$$\frac{x^5}{x^3} = x^{5-3} = x^2$$

Likewise, the expression $\frac{x^3}{x^5}$ simplifies to $\frac{1}{x^2}$, because $\frac{x^3}{x^5} = \frac{xxx}{xxxxx} = \frac{1}{x^2}$. Applying the Quotient Rule the same problem (subtracting exponents of the common base), would have resulted in a **negative** exponent.

$$\frac{x^3}{x^5} = x^{3-5} = x^{-2}$$

If $\frac{x^3}{x^5} = \frac{1}{x^2}$ and $\frac{x^3}{x^5} = x^{-2}$ then $x^{-2} = \frac{1}{x^2}$.

From this example you can see that a negative exponent and a positive exponent are **reciprocals**.

For any nonzero real number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

When a factor is moved from the numerator to the denominator (or from the denominator to the numerator) the sign on the exponent changes.

Example A: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ A common mistake is to confuse negative exponents with negative numbers. $3^{-2} \neq -9$

Example B: $x^{-2} = \frac{1}{x^2}$

Example C: $\frac{1}{y^{-5}} = y^5$

Try problems 1 - 14. Write the following with **positive** exponents only.

1. $x^{-3} =$

2. $5^{-2} =$

3. $2^{-3} =$

4. $2^{-5} =$

5. $y^{-6} =$

6. $m^{-1} =$

7. $(-6)^{-2} =$

8. $(-2)^{-2} =$

9. $\frac{1}{y^{-3}} =$

10. $\frac{1}{m^{-7}} =$

11. $\frac{1}{5^{-2}} =$

12. $\frac{1}{(-2)^{-3}}$

13. $\frac{1}{(-3)^{-2}} =$

14. $\frac{1}{(-4)^{-2}} =$

** Check your answers against the key before you continue. **

If a quotient is raised to a negative exponent invert the fraction and change the sign on the exponent to positive: $\left(\frac{a}{b}\right)^{-3} = \left(\frac{b}{a}\right)^{3}$.

Example D: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{3^2}{2^2} = \frac{9}{4}$

Example E: $\left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^{3} = \frac{y^3}{x^3}$

Example F: $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^{5} = \frac{2^5}{1^5} = 2^5 = 32$

Try problems 15 - 21:

15. $\left(\frac{5}{2}\right)^{-2}$ =

16. $\left(\frac{3}{5}\right)^{-2}$ =

17. $\left(\frac{5}{4}\right)^{-2}$ =

18. $\left(\frac{5}{3}\right)^{-1}$ =

19. $\left(\frac{3}{7}\right)^{-1}$ =

20. $\left(\frac{3}{2}\right)^{-3}$ =

** Check your answers against the key before you continue. **

Example G: $(3x)^{-1} = \frac{1}{(3x)^1} = \frac{1}{3x}$ Note: The base for the negative exponent is $(3x)$.

Example H: $3x^{-1} = 3 \cdot \frac{1}{x^1} = \frac{3}{x}$ Note: The base for the negative exponent is x, not $3x$.

Try problems 21 - 26.

21. $(5m)^{-1}$ =

22. $5m^{-1}$ =

23. $(2z)^{-1}$ =

24. $8b^{-2}$ =

25. $(8c)^{-1}$ =

26. $3t^{-1}$ =

** Check your answers against the key before you continue. **

Since exponential expression with negative exponents can be re-written with positive exponents the rules for exponents, are true for **negative** exponents as well.