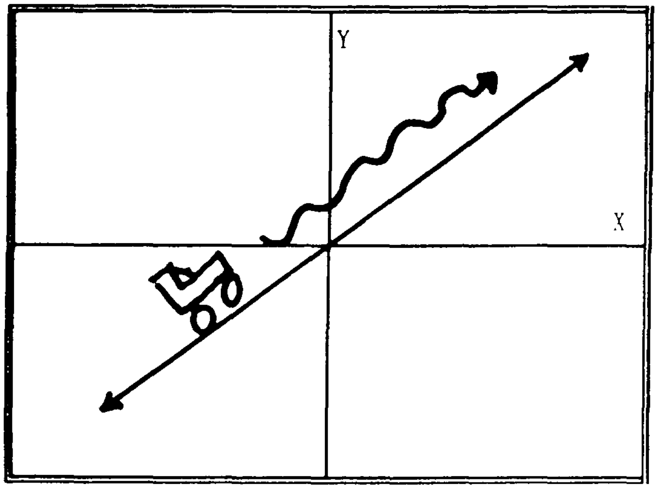
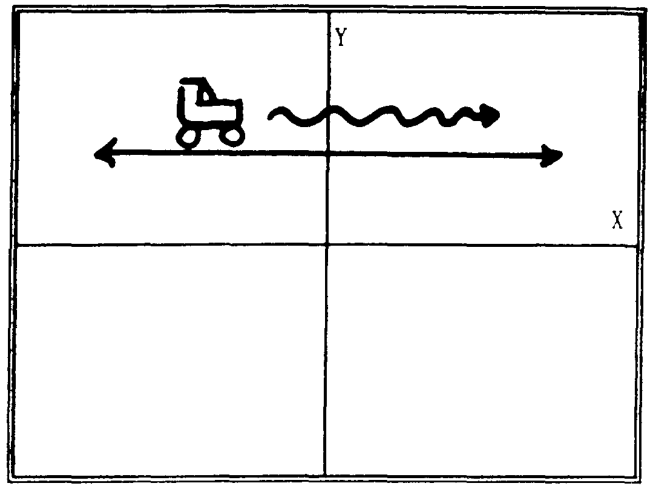
SLOPE

MAT 0024 BASIC ALGEBRA

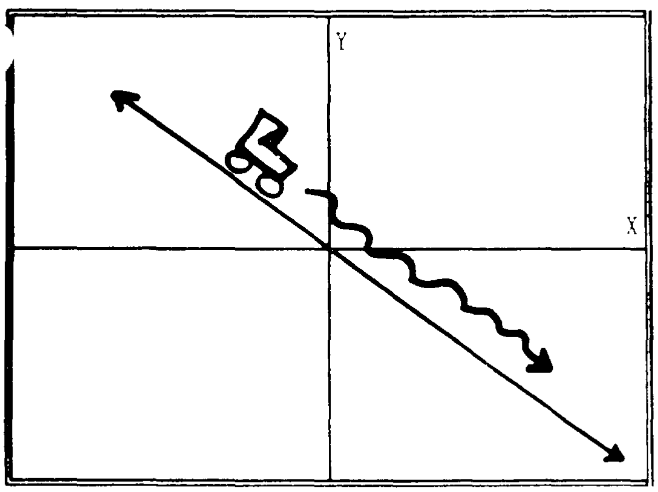
Positive Slope



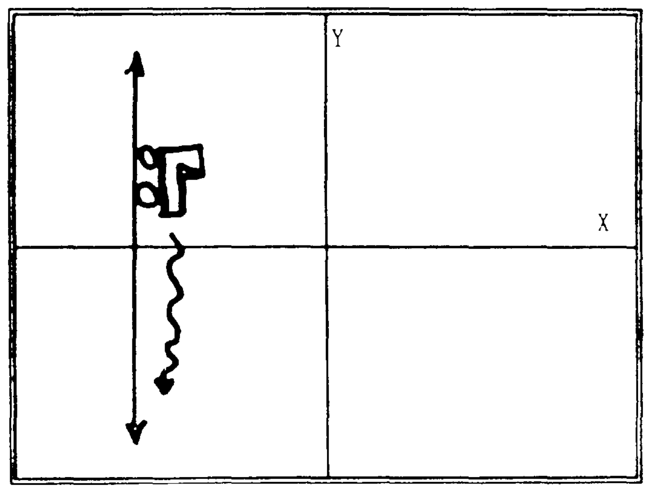
Zero Slope $y = k$



Negative Slope



Undefined Slope $x = k$



Formulas to find the slope

To find the slope of a line through the points (x_1, y_1) and (x_2, y_2) use the Slope Formula.

$$m = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1}$$

To find the slope of a line from its equation use the Slope-Intercept Form.

$$y = mx + b$$

The slope is m, the y-intercept is b.

The **slope** of a line is the measure of the steepness of the line. Given two points on a line, the slope is the **ratio** of the vertical change (**rise** between the points) and the horizontal change (**run** between the points).

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

<u>EXAMPLE 1</u>: Find the slope of a line containing the points (3,2) and (5,7). Use the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_2}$.

Let (3,2) = (x_1, y_1) and (5,7) = (x_2, y_2): $x_1 = 3$: $y_1 = 2$; $x_2 = 5$; $y_2 = 7$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 2}{5 - 3} = \frac{5}{2}$$

<u>EXAMPLE 2</u>: Find the slope of a line containing the points (5,-7) and (-3,6).
Let (5,-7) = (x_1, y_1) and (-3,6) = (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-7)}{-3 - 5} = \frac{13}{-8} = -\frac{13}{8}$$

<u>EXAMPLE 3</u>: Find the slope of a line $x = 5$. Find two points (solutions) to the equation $x = 5$.
Let (5,1) = (x_1, y_1) and (5,2) = (x_2, y_2).

x	y
5	1
5	2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{5 - 5} = \frac{1}{0}$$

Division by zero is undefined. The slope of the line $x = 5$ is **undefined**.
It is true that, in general, **the slope of a vertical line is undefined**.

<u>EXAMPLE 4</u>: Find the slope of a line $y = 3$ Find two points (solutions) to the equation $y = 3$.
Let (1,3) = (x_1, y_1) and (2,3) = (x_2, y_2).

x	y
1	3
2	3

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{2 - 1} = \frac{0}{1} = 0$$

The slope of the line $y = 3$ is zero.
It is true that, in general, **the slope of a horizontal line is zero**.

EXAMPLE 5: Find the slope of the line represented by the equation $y = 2x + 5$.
When a linear equation in two variables is written in **slope-intercept form**,

$$y = mx + b$$

the **m** is the slope of the line and the **b** is the *y*-intercept. Since the equation $y = 2x + 5$ is in slope-intercept form, the slope (**m**) = 2, and 5 is the *y*-intercept.

EXAMPLE 6: Find the slope and the *y*-intercept of the line whose equation is $3x - 2y = 4$.
Write the equation in slope-intercept form, $y = mx + b$.

$$3x - 2y = 4 \qquad \text{Subtract 3x from each side.}$$

$$-2y = -3x + 4$$

$$\frac{-2y}{-2} = \frac{-3}{-2}x + \frac{4}{-2} \qquad \text{Divide by } -2.$$

$$y = \frac{3}{2}x - 2$$

The slope (**m**) = $\frac{3}{2}$, the *y*-intercept (**b**) is -2.

EXAMPLE 7: Find the slope and the *y*-intercept of the line whose equation is $y = 5x$. Write the equation in slope-intercept form, $y = mx + b$.

$$y = 5x \qquad \text{For the missing "b", add zero.}$$

$$y = 5x + 0$$

The slope (**m**) = 5, the *y*-intercept (**b**) is 0.

EXERCISES

Problems 1 - 20: Find the slope of the line which contains each of the following pairs of points.

1. (2,4), (4, 5)	2. (1,5), (3,8)	3. (3,1), (1,5)	4. (3,6), (1,3)
5. (-1,4),(2,-2)	6. (-3,2),(1,-6)	7. (3,-2), (-3,2)	8. (5,-4), (1,-2)
9. (-4,-2), (-8, 2)	10. (-5,-2), (-1,-8)	11. (0,3), (4,6)	12. (6,0), (4,-2)
13. (-2,-4), (6,0)	14. (-7,-2), (-2,0)	15. (3,5),(-1,5)	16. (4,8),(-3,8)

17. (2,6), (2,3)   18. (-3,4), (-3,-1)   19. (3,6), (-4,6)   20. (5,-2),(5,4)

oblems 21 - 40: Find the slope and the *y*-intercept of the line represented by each of the following equations. If there is no *y*-intercept, write **no *y*-intercept**.

21. $3x + y = 15$ 22. $2x - y = 9$ 23. $3x - y = 7$ 24. $2x - y = 11$

25. $2x + 3y = 9$ 26. $3x + 2y = 12$ 27. $5x - 3y = 15$ 28. $2x - 5y = 10$

29. $8x + 4y = 13$ 30. $6x + 4y = 9$ 31. $4x - 8y = 11$ 32. $6x - 3y = 9$

33. $x = 5$ 34. $y = 4$ 35. $y = -8$ 36. $x = -1$

ANSWER KEY

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. -2   4. $\frac{3}{2}$ 5. -2   6. -2   7. $-\frac{2}{3}$

$-\frac{1}{2}$ 9. -1   10. $-\frac{3}{2}$ 11. $\frac{3}{4}$ 12. 1   13. $\frac{1}{2}$ 14. $\frac{2}{5}$

15. 0   16. 0   17. undefined   18. undefined   19. 0   20. undefined

21. $m = -3$, *y*-intercept = 15   22. $m = 2$, *y*-intercept = -9   23. $m = 3$, *y*-intercept = -7

24. $m = 2$, *y*-intercept = -11   25. $m = -\frac{2}{3}$, *y*-intercept = 3   26. $m = -\frac{3}{2}$, *y*-intercept = 6

27. $m = \frac{5}{3}$, *y*-intercept = -5   28. $m = \frac{2}{5}$, *y*-intercept = -2   29 $m = -2$, *y*-intercept = $\frac{13}{4}$ *or* $3\frac{1}{4}$

30. $m = -\frac{3}{2}$, *y*-intercept = $\frac{9}{4}$ *or* $2\frac{1}{4}$ 31 $m = \frac{1}{2}$, *y*-intercept = $-\frac{11}{8}$ *or* $-1\frac{3}{8}$

32. $m = 2$, *y*-intercept = -3   33. m = undefined, no *y*-intercept (Why not?)

34. $m = 0$, *y*-intercept = 4   35. $m = 0$, *y*-intercept = -8

36. m = undefined, no *y*-intercept

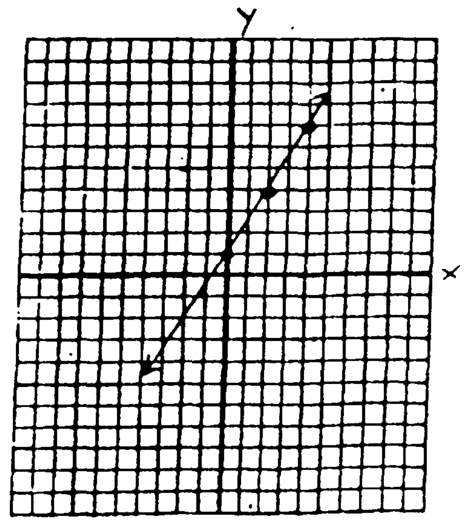
If an equation is written in the form $y = mx + b$, the m (the coefficient of x) is the **slope**, and the b is the **y-intercept** (where the graph of the line crosses the y-axis).

EXAMPLE 1: Graph $y = \frac{3}{2}x + 1$

Since the equation is in slope-intercept form (solved for y), the

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y \text{ (vertical change)}}{\text{change in } x \text{ (horizontal change)}} = \frac{3}{2}$$

The b (the y-intercept) in the above example is 1. This implies that the graph of this line crosses the y-axis at (0,1).



To graph this line, **first** plot the y-intercept at (0,1). (0,1) is a solution for this equation and the first point you will plot. To find a **second** solution, from (0,1) move up 3 units (rise) and move 2 units to the right (run). You are now at (2,4) - plot this point on the graph. To find a **third** solution, from (2, 4) again move up 3 units, and 2 units to the right. You are now at (4,7) - plot this point on the graph.

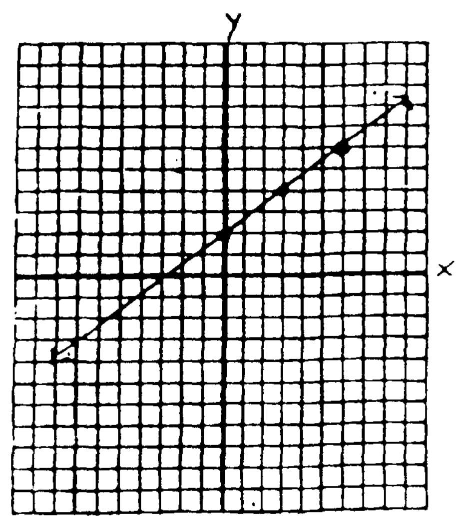
A line drawn through these points represents the graph of the equation $y = \frac{3}{2}x + 1$.

EXAMPLE 2: Graph $3y - 2x = 6$

The equation must be in slope-intercept form (solved for y).

$$3y - 2x = 6$$
$$3y = 2x + 6$$
$$\frac{3y}{3} = \frac{2x}{3} + \frac{6}{3}$$
$$y = \frac{2}{3}x + 2$$

The slope (m) is $\frac{2}{3}$ The y-intercept is 2.



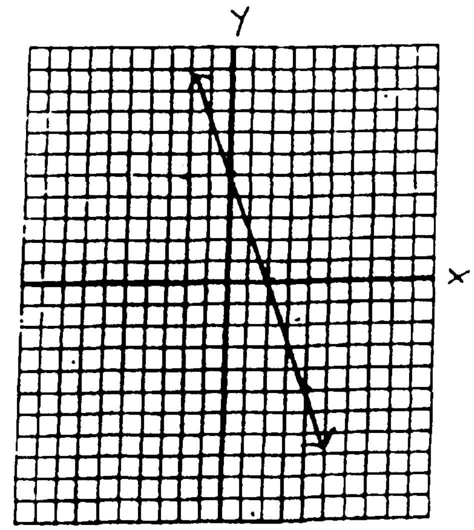
To graph this line, **first** plot the y-intercept at (0,2). (0,2) is a solution for this equation and the first point you will plot. To find a second solution, from (0,2) move up 2 units and then move 3 units to the right. You are now at (3,4) - plot this point on the graph. To find a third solution, from (3,4) move up 2 units and then move 3 units to the right. You are now at (6,6) - plot this point on the graph.

A line drawn through these points represents the graph of the equation $3y - 2x = 6$.

EXAMPLE 3: Graph $5x + 2y = 10$

The equation must be in slope-intercept form (solved for *y*).

$$5x + 2y = 10$$
$$2y = -5x + 10$$
$$\frac{2y}{2} = \frac{-5x}{2} + \frac{10}{2}$$
$$y = -\frac{5}{2}x + 5$$



The slope (*m*) is $-\frac{5}{2}$ ↑ ↑ The *y*-intercept is 5.

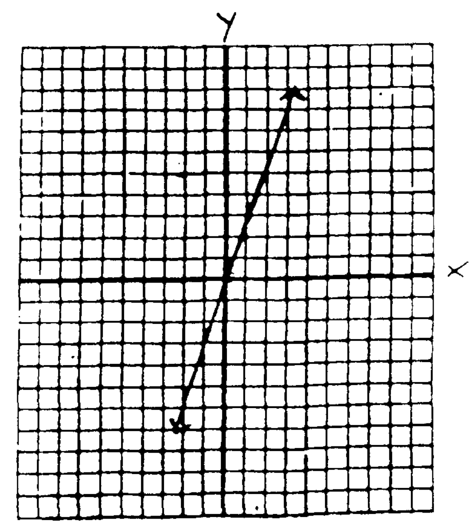
To graph this line, **first** plot the *y*-intercept at (0,5). (0,5) is a solution for this equation and the first point you will plot. To find a second solution, from (0,5) move **down** (the slope is **negative**) 5 units and then move 2 units to the right. You are now at (2,0) - plot this point on the graph. To find a third solution, from (2,0) move down 5 units and then move 2 units to the right. You are now at (4,-5) - plot this point on the graph.

A line drawn through these points represents the graph of the equation $5x + 2y = 10$.

XAMPLE 4: Graph $y = 3x$

For the above equation to be in the form $y = mx + b$, add "0".

$$y = 3x + 0$$



The slope (*m*) is $\frac{3}{1}$ ↑ ↑ The *y*-intercept is 0.

To graph this line, **first** plot the *y*-intercept at (0,0). (0,0) is a solution for this equation and the first point you will plot. To find a second solution, from (0,0) move up 3 units and then move 1 unit to the right. You are now at (1,3) - plot this point on the graph. To find a third solution, from (1,3) move up 3 units and then move 1 unit to the right. You are now at (2,6) - plot this point on the graph.

A line drawn through these points represents the graph of the equation $y = 3x$.

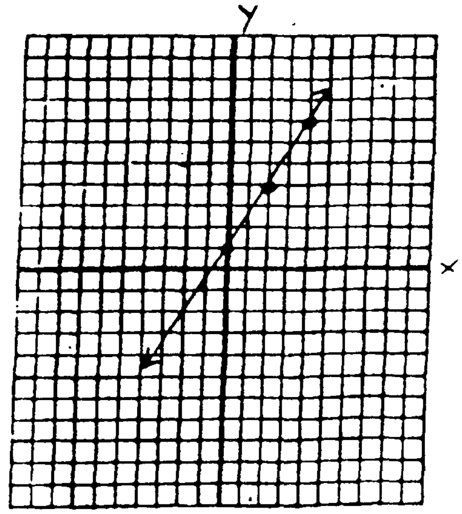
If an equation is written in the form $y = mx + b$, the m (the coefficient of x) is the **slope**, and the b is the ***y*-intercept** (where the graph of the line crosses the y-axis).

EXAMPLE 1: Graph $y = \frac{3}{2}x + 1$

Since the equation is in slope-intercept form (solved for y), the

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y \text{ (vertical change)}}{\text{change in } x \text{ (horizontal change)}} = \frac{3}{2}$$

The b (the y-intercept) in the above example is 1. This implies that the graph of this line crosses the y-axis at (0,1).



To graph this line, **first** plot the y-intercept at (0,1). (0,1) is a solution for this equation and the first point you will plot. To find a **second** solution, from (0,1) move up 3 units (rise) and move 2 units to the right (run). You are now at (2,4) - plot this point on the graph. To find **a third** solution, from (2, 4) again move up 3 units, and 2 units to the right. You are now at (4,7) - plot this point on the graph.

A line drawn through these points represents the graph of the equation $y = \frac{3}{2}x + 1$.

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The equation must be in slope-intercept form (solved for y).

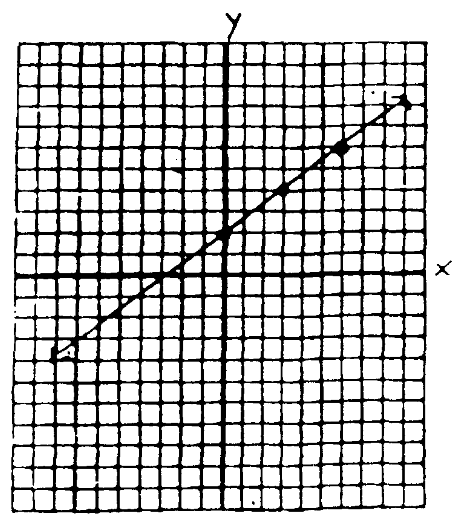
$$3y - 2x = 6$$

$$3y = 2x + 6$$

$$\frac{3y}{3} = \frac{2x}{3} + \frac{6}{3}$$

$$y = \frac{2}{3}x + 2$$

The slope (m) is $\frac{2}{3}$ The y-intercept is 2.



To graph this line, **first** plot the y-intercept at (0,2). (0,2) is a solution for this equation and the first point you will plot. To find a second solution, from (0,2) move up 2 units and then move 3 units to the right. You are now at (3,4) - plot this point on the graph. To find a third solution, from (3,4) move up 3 units and then move 2 units to the right. You are now at ~~(6,6)~~ (5,7) - plot this point on the graph.

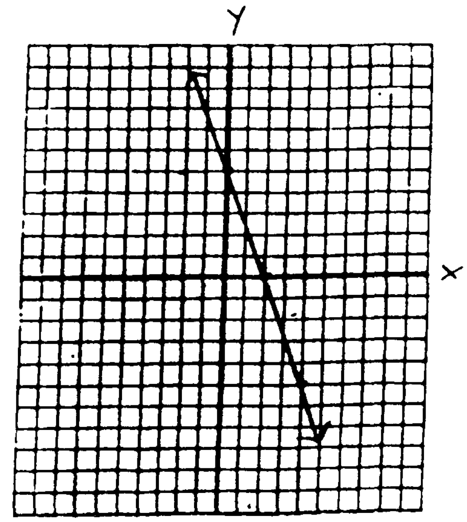
A line drawn through these points represents the graph of the equation $3y - 2x = 6$.

EXAMPLE 3: Graph $5x + 2y = 10$

The equation must be in slope-intercept form (solved for y).

$$5x + 2y = 10$$
$$2y = -5x + 10$$
$$\frac{2y}{2} = \frac{-5x}{2} + \frac{10}{2}$$
$$y = -\frac{5}{2}x + 5$$

The slope (m) is $-\frac{5}{2}$ ↑ ↑ The y-intercept is 5.



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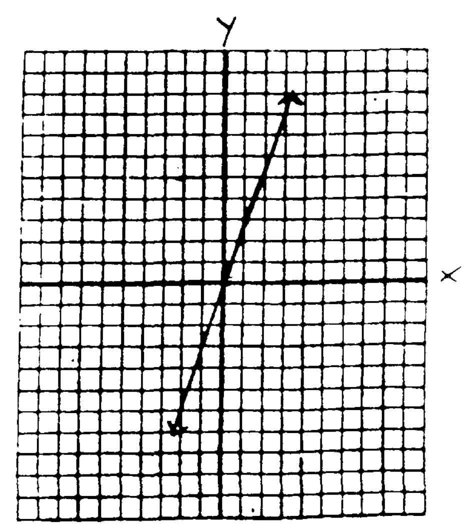
A line drawn through these points represents the graph of the equation $5x + 2y = 10$.

XAMPLE 4: Graph $y = 3x$

For the above equation to be in the form $y = mx + b$, add "0".

$$y = 3x + 0$$

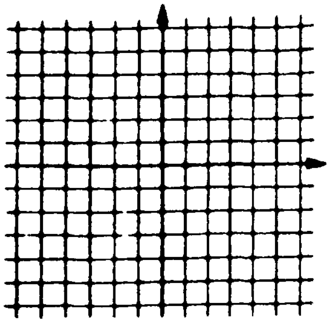
The slope (m) is $\frac{3}{1}$ ↑ ↑ The y-intercept is 0.



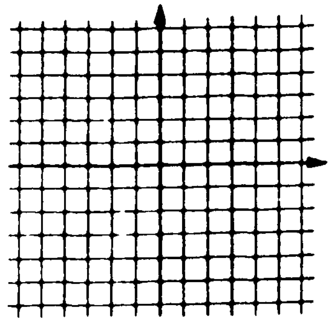
To graph this line, **first** plot the y-intercept at (0,0). (0,0) is a solution for this equation and the first point you will plot. To find a second solution, from (0,0) move up 3 units and then move 1 unit to the right. You are now at (1,3) - plot this point on the graph. To find a third solution, from (1,3) move up 3 units and then move 1 unit to the right. You are now at (2,6) - plot this point on the graph.

A line drawn through these points represents the graph of the equation $y = 3x$.

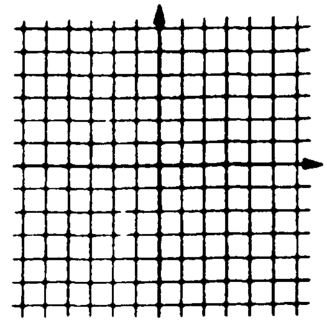
1) $y = 2x + 1$



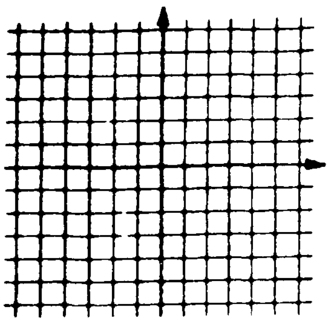
2) $y = 2x - 4$



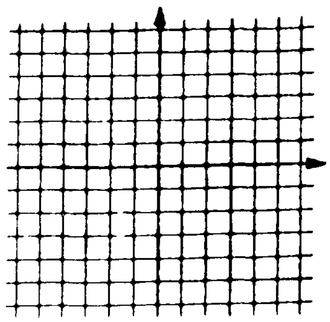
3) $y = -2x + 4$



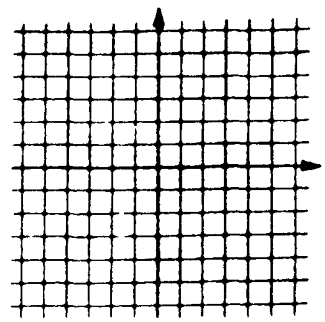
4) $y = -2x$



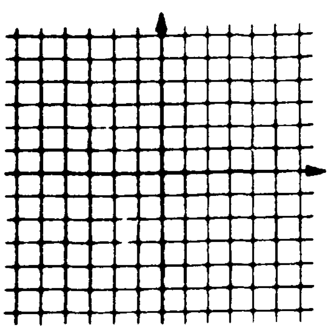
5) $y = 3x - 3$



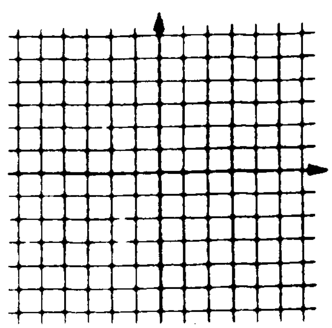
6) $y = -4x + 2$



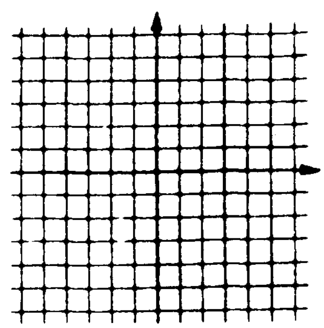
7) $x + 2y = -6$



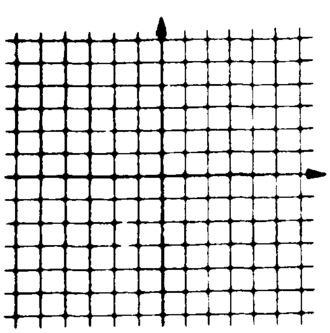
8) $x - 2y = 6$



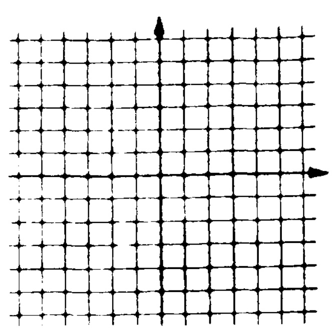
9) $2x - 3y = -6$



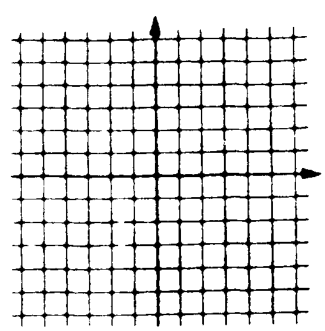
10) $x - 2y = -6$



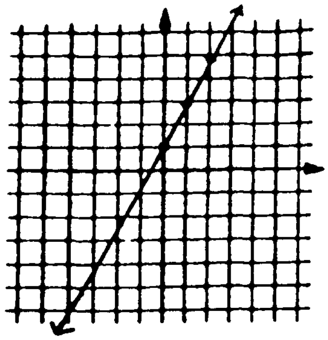
11) $3x + 2y = -6$



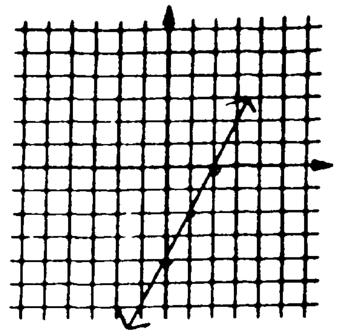
12) $-3x + y = 6$



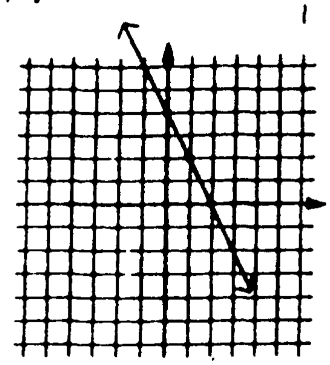
1) $y = 2x + 1$ $m = \frac{2}{1}$



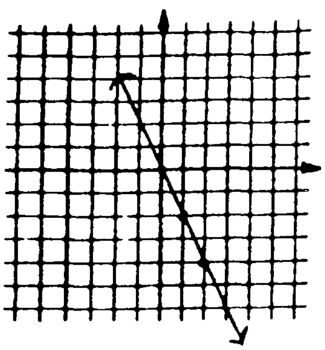
2) $y = 2x - 4$ $m = \frac{2}{1}$



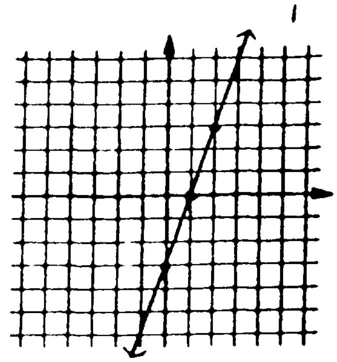
3) $y = -2x + 4$ $m = \frac{-2}{1}$



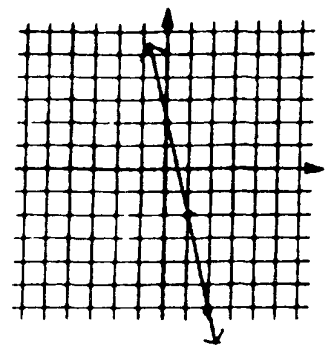
4) $y = -2x$ $m = -\frac{2}{1}$



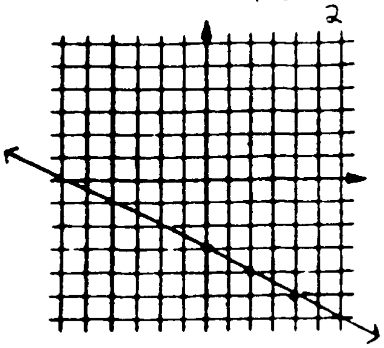
5) $y = 3x - 3$ $m = \frac{3}{1}$



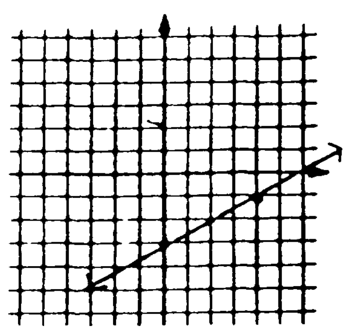
6) $y = -4x + 2$ $m = -\frac{4}{1}$



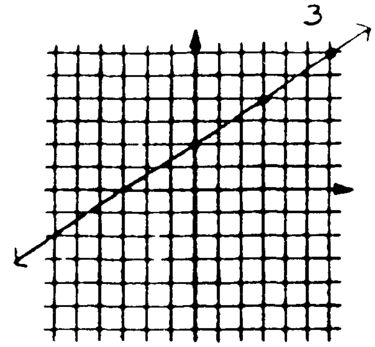
7) $x + 2y = -6$ $m = -\frac{1}{2}$



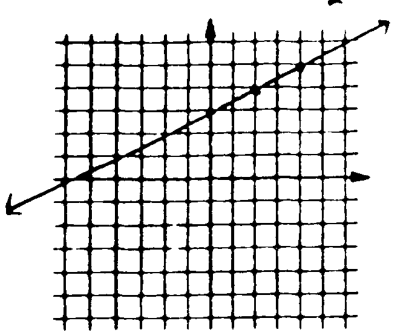
8) $x - 2y = 6$ $m = \frac{1}{2}$



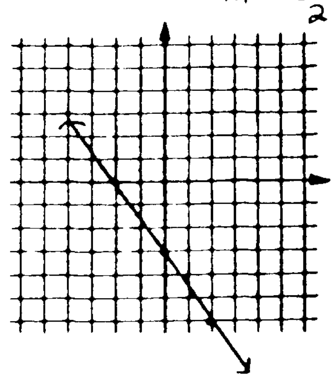
9) $2x - 3y = -6$ $m = \frac{2}{3}$



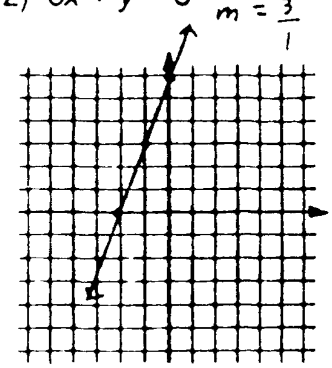
10) $x - 2y = -6$ $m = \frac{1}{2}$



11) $3x + 2y = -6$ $m = \frac{-3}{2}$



12) $-3x + y = 6$ $m = \frac{3}{1}$



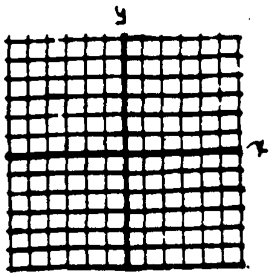
MAT 0024L

GRAPHING

Complete the tables of values and graph the lines:

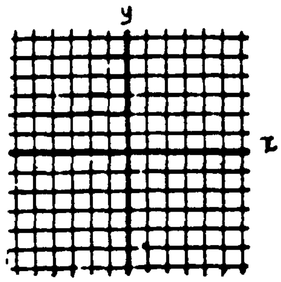
1. $y = 2x$

x	y
0	
1	
2	
-1	



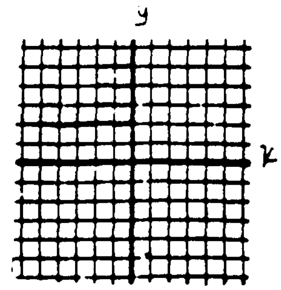
2. $y = 2x - 4$

x	y
0	
1	
3	
4	



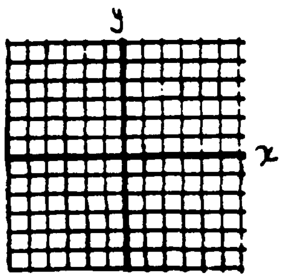
3. $y = -2x + 4$

x	y
0	
1	
2	
-2	



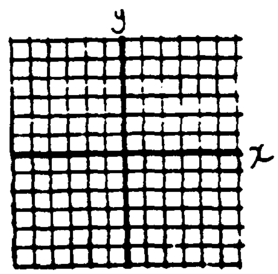
4. $y = -2x$

x	y



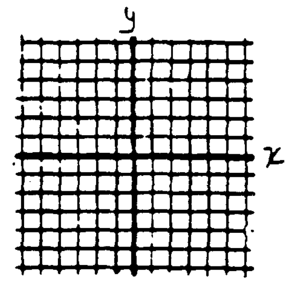
5. $y = 3x - 3$

x	y



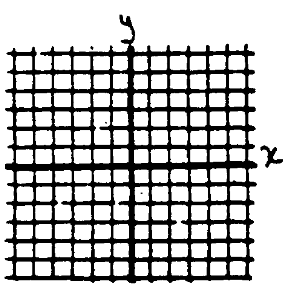
6. $y = {}^{-}4x + 2$

x	y



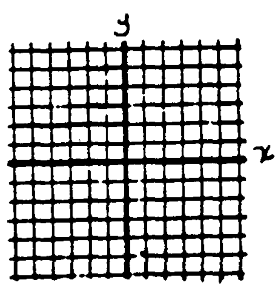
7. $x + 2y = 6$

x	y
0	
	0



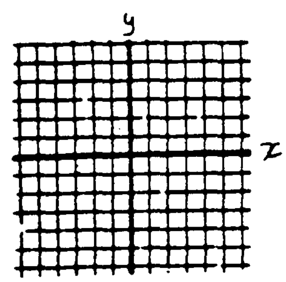
8. $x - 2y = 6$

x	y
0	
	0



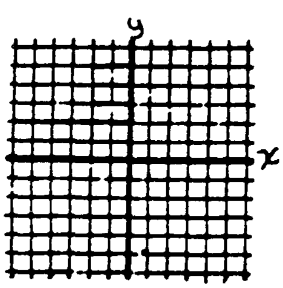
9. $2x - 3y = -6$

x	y
0	
	0



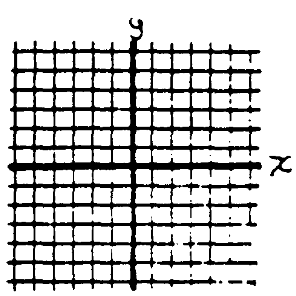
10. $x - 2y = -6$

x	y



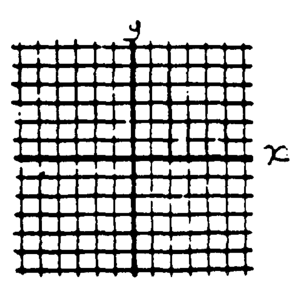
11. $3x + 2y = -6$

x	y

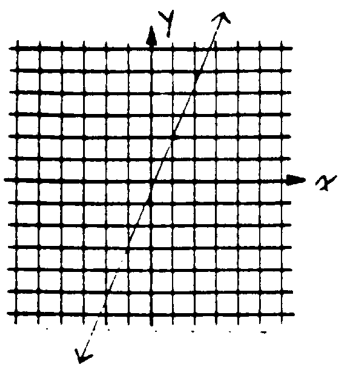


12. $-3x + y = 6$

x	y



1.



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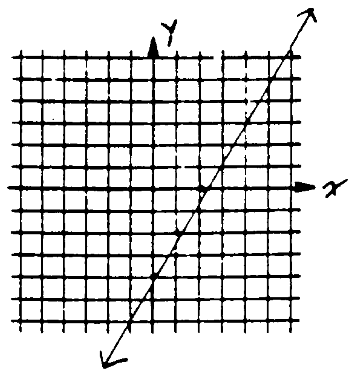

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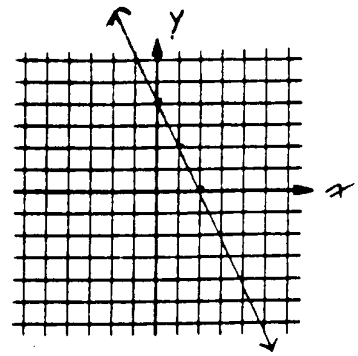

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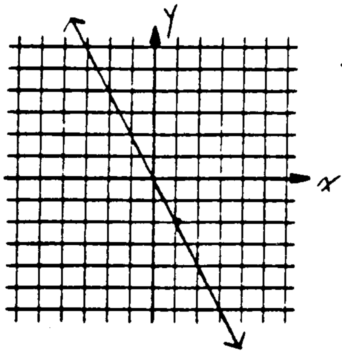

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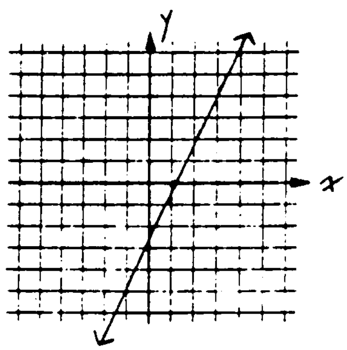

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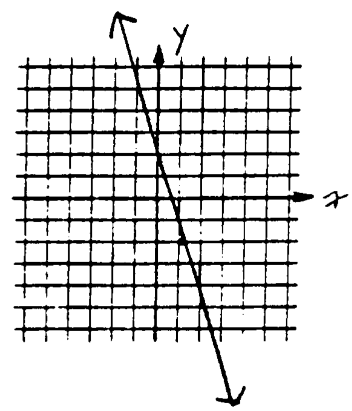

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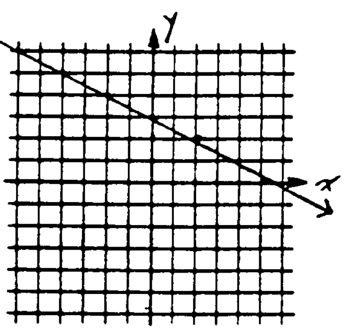

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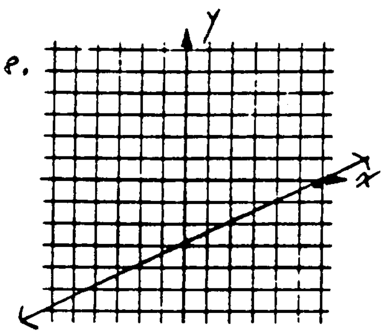
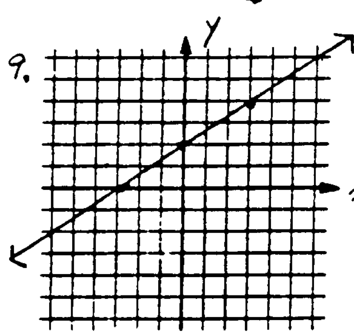
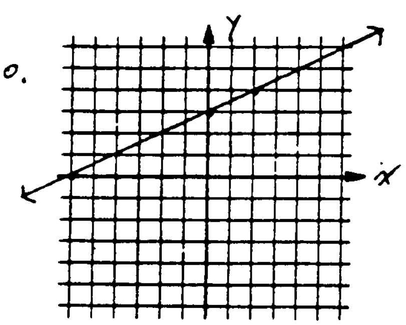


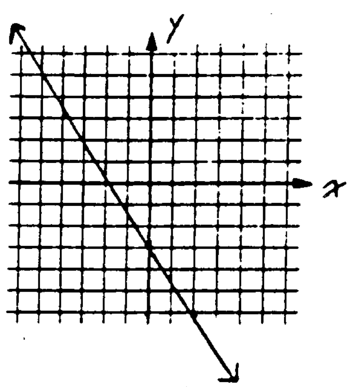
9.



10.



11.



12.