NUMBER PROPERTIES

MAT0024 **Basic Algebra Lab**

I. COMMUTATIVE PROPERTY

The order may be changed without affecting the result. (This is like **commuting** to and from work or school.)

A. Commutative Property of **Addition**: $a + b = b + a$
Example: $3 + 4 = 4 + 3$
Example: $7(6 + 9) = 7(9 + 6)$
Example: $1 + (2 + 3) = 1 + (3 + 2)$
Example: $1 + (2 + 3) = (2 + 3) + 1$
Example: $(4x + 7y) + 10 = (7y + 4x) + 10$

B. Commutative Property of **Multiplication**: $a \cdot b = b \cdot a$
Example: $(3) \cdot (4) = (4) \cdot (3)$
Example: $7 \cdot (6 + 9) = (6 + 9) \cdot 7$
Example: $1 \cdot (2 \cdot 3) = 1 \cdot (3 \cdot 2)$
Example: $1 \cdot (2 \cdot 3) = (2 \cdot 3) \cdot 1$
Example: $(4x \cdot 7y) + 10 = (7y \cdot 4x) + 10$

Subtraction and division are not commutative. Why not?

II. ASSOCIATIVE PROPERTY

The way in which the numbers are **associated** (by means of parentheses) may be changed without affecting the result. Notice that the order does not change.

A. Associative Property of **Addition**: $a + (b + c) = (a + b) + c$
Example: $1 + (2 + 3) = (1 + 2) + 3$
Example: $(5 + 7) + 1 = 5 + (7 + 1)$
Example: $(4x + 7y) + 10 = 4x + (7y + 10)$

B. Associative Property of **Multiplication**: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Example: $1 \cdot (2 \cdot 3) = (1 \cdot 2) \cdot 3$
Example: $(5 \cdot 7) \cdot 1 = 5 \cdot (7 \cdot 1)$
Example: $(4x \cdot 7y) \cdot 10 = 4x \cdot (7y \cdot 10)$

Do you think subtraction and division are associative?

III. DISTRIBUTIVE PROPERTY

A multiplier **distributes** over a sum or difference. Observe that this property is often given "backwards": $a \cdot b + a \cdot c = a \cdot (b + c)$

Example: $4 \cdot (x + 7) = 4 \cdot x + 4 \cdot 7$ *or* $4(x + 7) = 4x + 28$
Example: $7 \cdot (y - 9) = 7 \cdot y - 7 \cdot 9$ *or* $7 (y - 9) = 7y - 63$
Example: $3 \cdot (4x + 7y - 6) = 12x + 21y - 18$

Example: $4x + 8 = 4 \cdot (x + 2)$
Example: $6y - 54 = 6 \cdot (y - 9)$

IV.A. IDENTITY ELEMENT FOR ADDITION
ADDITIVE IDENTITY
ZERO PROPERTY FOR ADDITION

For every x in a set, $x + 0 = x$ and $0 + x = x$.

This property is so named because when zero is added to any number, the result is **identically** the same as the original number.

Example: $6 + 0 = 6$ *or* $0 + 6 = 6$
Example: $a + 0 = a$ *or* $0 + a = a$
Example: $(b + 7) + 0 = (b + 7)$

IV.B. IDENTITY ELEMENT FOR MULTIPLICATION
MULTIPLICATIVE IDENTITY

For every x in a set, $x \cdot 1 = x$ *and* $1 \cdot x = x$.

When a number is multiplied by one the result is **identically** the same as the original number.

Example: $7 \cdot 1 = 7$ *or* $1 \cdot 7 = 7$
Example: $a \cdot 1 = a$ *or* $1 \cdot a = a$
Example: $(4x - 3) \cdot 1 = (4x - 3)$

V.A. ADDITIVE INVERSE PROPERTY
INVERSE PROPERTY FOR ADDITION

$x + (-x) = 0$

Every number in the set has an inverse $(-x)$ such that the sum of the number and its inverse is the identity zero.

Example: $6 + (-6) = 0$
Example: $(-49) + 49 = 0$
Example: $8a + (-8a) = 0$

V. . MULTIPLICATIVE INVERSE PROPERTY
INVERSE PROPERTY FOR MULTIPLICATION

$x \cdot \frac{1}{x} = 1$

For every number in the set (except zero) there is an inverse such that the product of the number and its inverse is equal to the identity one.

Example: $4 \cdot \frac{1}{4} = 1$

Example: $\frac{1}{6} \cdot 6 = 1$

Example: $(-5) \cdot \left(-\frac{1}{5}\right) = 1$

Example: $\left(\frac{3}{2}\right) \cdot \left(\frac{2}{3}\right) = 1$

Example: $\left(-\frac{5}{7}\right)\left(-\frac{7}{5}\right) = 1$

Zero has no multiplicative inverse. Why not?

Notice that for the identity properties, the result is identically the same. For the inverse properties, the result is always the identity (zero for addition, or one for multiplication.)

EXERCISES

ive the complete name of the property used:

1. $3 + x = x + 3$ — 1. ______________________

2. $3 \cdot (x + 5) = (x + 5) \cdot 3$ — 2. ______________________

3. $3 \cdot (x + 5) = 3 \cdot (5 + x)$ — 3. ______________________

4. $3 \cdot (x + 5) = 3 \cdot x + 3 \cdot 5$ — 4. ______________________

5. $3 \cdot x + 3 \cdot 5 = 3 \cdot (x + 5)$ — 5. ______________________

6. $3 \cdot (5 \cdot x) = (3 \cdot 5) \cdot x$ — 6. ______________________

7. $6 + (4 + x) = (6 + 4) + x$ — 7. ______________________

8. $6 + 0 = 6$ — 8. ______________________

9. $8 \cdot 1 = 8$ — 9. ______________________

10. $8 + 0 = 8$ — 10. ______________________

11. $8 + (-8) = 0$ — 11. ______________________

12. $8 \cdot \frac{1}{8} = 1$ — 12. ______________________

13. $x \cdot 1 = x$ — 13. ______________________

14. $x + (-x) = 0$ — 14. ______________________

15. $8 \cdot (x \cdot 1) = 8 \cdot x$ — 15. ______________________

16. $8 \cdot (x \cdot 1) = (8 \cdot x) \cdot 1$ — 16. ______________________

17. $8 \cdot (x \cdot 1) = 8 \cdot (1 \cdot x)$ — 17. ______________________

18. $8 \cdot [x + (-x)] = 8 \cdot 0$ — 18. ______________________

19. $8 \cdot [x + (-x)] = 8 \cdot [(-x) + x]$ — 19. ______________________

20. $8 \cdot [x + (-x)] = [x + (-x)] \cdot 8$ — 20. ______________________

21. $8 \cdot (6 + 0) = 8 \cdot 6$ — 21. ______________________

22. $8 + (6 + 0) = 8 + 6$ — 22. ______________________

23. $8 + (6 + 0) = 8 + (0 + 6)$ — 23. ______________________

EXERCISES

Key

Give the complete name of the property used:

	$3 + x = x + 3$	1.	Commutative, Add.
2.	$3 \cdot (x + 5) = (x + 5) \cdot 3$	2.	" , Mult.
3.	$3 \cdot (x + 5) = 3 \cdot (5 + x)$	3.	" , Add.
4.	$3 \cdot (x + 5) = 3 \cdot x + 3 \cdot 5$	4.	Distributive
5.	$3 \cdot x + 3 \cdot 5 = 3 \cdot (x + 5)$	5.	"
6.	$3 \cdot (5 \cdot x) = (3 \cdot 5) \cdot x$	6.	Associative, Mult.
7.	$6 + (4 + x) = (6 + 4) + x$	7.	" , Add.
8.	$6 + 0 = 6$	8.	Identity, Add.
9.	$8 \cdot 1 = 8$	9.	" , Mult.
10.	$8 + 0 = 8$	10.	" , Add.
11.	$8 + (-8) = 0$	11.	Inverse, Add.
12.	$8 \cdot \frac{1}{8} = 1$	12.	Inverse, Mult.
1.	$x \cdot 1 = x$	13.	Identity, Mult.
14.	$x + (-x) = 0$	14.	Inverse, Add.
15.	$8 \cdot (x \cdot 1) = 8 \cdot x$	15.	Identity, Mult.
16.	$8 \cdot (x \cdot 1) = (8 \cdot x) \cdot 1$	16.	Associative, Mult.
17.	$8 \cdot (x \cdot 1) = 8 \cdot (1 \cdot x)$	17.	Commutative, Mult.
18.	$8 \cdot [x + (-x)] = 8 \cdot 0$	18.	Inverse, Add.
19.	$8 \cdot [x + (-x)] = 8 \cdot [(-x) + x]$	19.	Commutative, Add.
20.	$8 \cdot [x + (-x)] = [x + (-x)] \cdot 8$	20.	" , Mult.
21.	$8 \cdot (6 + 0) = 8 \cdot 6$	21.	Identity, Add.
22.	$8 + (6 + 0) = 8 + 6$	22.	" , Add.
23.	$8 + (6 + 0) = 8 + (0 + 6)$	23.	Commutative, Add.